The **ACT** Course Book

MATHEMATICS & SCIENCE

Instructor's Edition

SAMPLE

Focusing on the Individual Student

SUMMIT EDUCATIONAL GROUP

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CONTENTS

TEST-TAKING FUNDAMENTALS

About the ACT 2
Your Commitment 3
ACT Structure 4
Content 5
Scoring 6
Knowing Your Limits 8

MATH OVERVIEW

The Mathematics Test 12
Format 14
Question Difficulty 15
Working Through the Math Test 16
Attractors 17
Setting Your Goal 18
Problem-Solving Tools 20
Plugging In 22
Choosing Numbers 24
Guesstimating 26
Bridging the Gap 28
Using Your Calculator 30

INTEGRATING ESSENTIAL SKILLS

Multiples, Factors, & Divisibility 36
Fractions 38
Ratios 40
Proportions 42
Digits & Place Value 44
Percents 46
Charts & Graphs 50

SAMPLE

Number & Quantity Foundations 56
Algebra Foundations 60
Geometry Foundations – *xy*-Plane 64
Geometry Foundations – Angles 68
Geometry Foundations – Triangles 72
Geometry Foundations – Area & Volume 76
Statistics & Probability Foundations 80

Integrating Essential Skills Practice 87

NUMBER & QUANTITY

Number Properties 106
Imaginary & Complex Numbers 108
Translation 112
Matrices 116
Vectors 118

Number & Quantity Practice 123

ALGEBRA

Exponents 130
Roots 132
Algebraic Expressions 134
Equations 136
Graphs of Linear Equations 138
Inequalities 140
Graphs of Inequalities 142
Systems of Equations 144
Quadratic Equations & Expressions 148
Polynomials 150

Algebra Practice 157

SAMPLE

GEOMETRY

Vocabulary	170
Similar & Congruent Triangles	172
Right Triangles	174
SOH CAH TOA	176
Midpoint & Distance	178
Transformations	180
Conic Sections	182
Geometry Practice	189

FUNCTIONS

Functions	200
Graphs of Functions	204
Trigonometry Identities	208
Laws of Sines & Cosines	212
The Unit Circle	214
Graphs of Sine & Cosine	218
Sequences	220
Logarithms	222
Functions Practice	227

STATISTICS & PROBABILITY

Averages	240
Combinations & Permutations	242
Probability	244
Statistics & Probability Practice	249

SAMPLE

SCIENCE OVERVIEW

The Science Test 254
Format and Scoring 255
Attractors 256
Setting Your Goal 258
Working Through the Science Test 260
General Tips 261

SCIENCE

The Scientific Method 264
Data Representation Overview 266
Interpretation 268
Relationship 272
Additional Data 276
Connection 280
Research Summary Overview 284
Data 286
Experimental Design 288
Evaluation 292
Conflicting Viewpoints Overview 298
Detail 300
Perspective 302
Assessment 306

Science Foundations 312

Science Practice 317

ANSWER KEY 365

SAMPLE

Guesstimating

- ❑ Most figures on the ACT are drawn to scale. Therefore, a viable strategy for many geometry questions is to use the figure to guide your reasoning or narrow down answer choices.

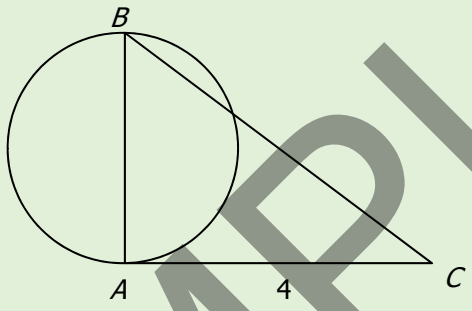
  Note that the ACT states that you should assume "illustrative figures are NOT necessarily drawn to scale." Usually, figures not drawn to scale will have a disclaimer stating so.

- ❑ Guesstimate on figures by comparing unknown values to known values. Use your answer sheet or pencil as a "ruler."

In the figure below, ~~if $\overline{AB}$ is a diameter of the circle and the circle has radius 1.5~~, what is the length of $\overline{BC}$?

A. 3
B. 4
C. 5
D. 7
E. 8

C



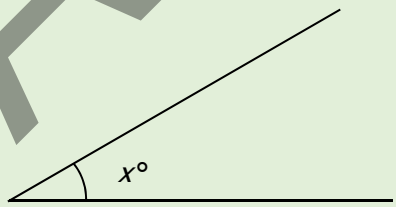
Can you eliminate answer choices logically?

Remember, the 90° angle is the largest, so it must be opposite the longest side, and the longest side must be greater than the sum of the two smaller sides

- ❑ Draw 90° and 45° angles to help you guesstimate angle measures.

Guesstimate the measure of the angle below.

$x =$ ~30



- ❑ Know the following approximations to help you guesstimate lengths.

$\sqrt{2} \approx 1.4$ $\sqrt{3} \approx 1.7$ $\pi \approx 3.1$

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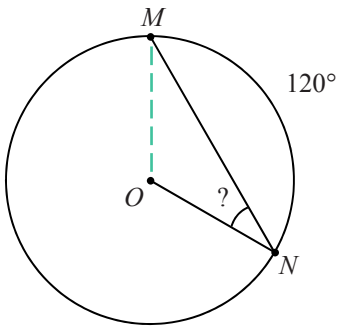
PUT IT TOGETHER

Guesstimate to solve the following:

1. In the figure below, the circle has center O and the measure of $\overset{\frown}{MN}$ is 120°. What is the measure of $\angle MNO$?



A. 30°
B. 40°
C. 45°
D. 60°
E. 80°

Guesstimate, or use geometry rules to solve.

Draw a line segment between points O and M to create $\triangle MON$. If arc $MN = 120°$, $\angle MON = 120°$. Since legs OM and ON are both radii of the circle, it is isosceles and $\angle N = \angle M$. If $\angle N = \angle M = x$, then $120 + 2x = 180$ and $x = 30°$.

2. Of the 398 seniors in a certain high school, approximately 51% are in the art club. Of the seniors who are in the art club, approximately 97% paint with acrylics. Which of the following is closest to the number of seniors who paint with acrylics?

F. 75
G. 100
H. 150
J. 200
K. 400

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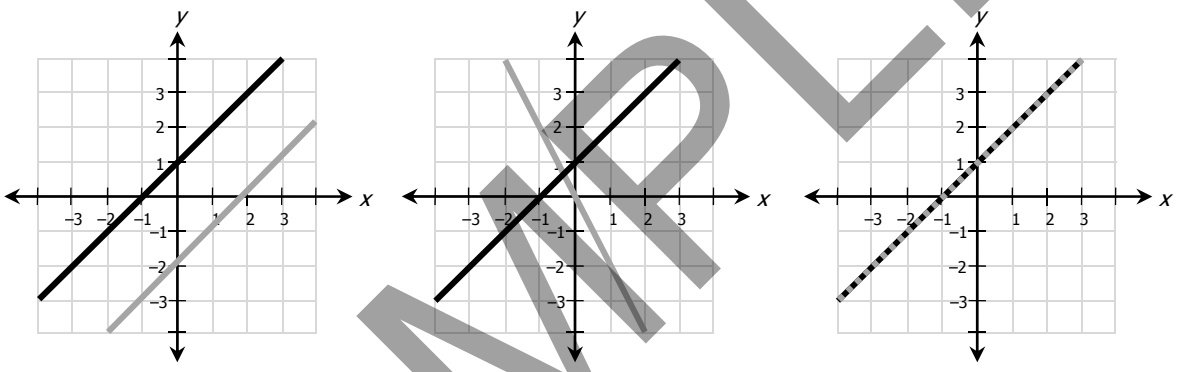
Systems of Equations

0-2 per test E M H

On the ACT, simultaneous equation questions ask you to find a mutual solution for two linear equations. When simultaneous equations are graphed, the mutual solution is the point of intersection.

Some simultaneous equation questions will give the two equations and ask you to solve. Other times, the equations are hidden in a word problem, requiring you to set up the equations first and then solve.

- A system of linear equations, also called simultaneous equations, is a set of two or more equations working together. Simultaneous equations can be solved graphically and algebraically. A system of two linear equations can have no solution, 1 solution, or infinitely many solutions.



No Solution | One Solution | Infinite Solutions

- **No Solutions** – When lines are parallel, there is no common solution. What does this mean algebraically? If we consider the slope-intercept forms of the two lines ($y = mx + b$), it means that the slopes (m) are the same and the y-intercepts (b) are different.

- **Infinitely Many Solutions** – When lines overlap, there are infinitely many solutions. Algebraically, this means that the two lines have the same slopes (m) and the same y-intercepts (b).

$2x + y = 5$
$4x + 2y = 10$

How many solutions does the pair of linear equations shown above have? infinite

How do you know?

Divide second equation by 2 to see they are the same line.

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❑ **Elimination Method** – Add or subtract equations to cancel one of the variables and solve for the other. You may have to multiply an equation by some number to eliminate a variable before the equations are added or subtracted.

If $2x + y = 16$ and $x - 2y = 3$, what is the value of x?

Stack the equations: $2x + y = 16$
$x - 2y = 3$

What do you need to multiply the top equation by to make the y disappear when you add the two equations? 2

Rewrite the equations and add them. Solve for x.

$x = 7$

❑ **Substitution Method** – Solve one equation for one of the variables, and then substitute that value for that variable in the other equation.

If $2x + y = 3$ and $-x - 3y = 6$, what is the value of x?

Solve the equation $2x + y = 3$ for y: $y = -2x + 3$

Substitute that value for y in the other equation: $-x - 3($ $-2x + 3$ $) = 6$

Solve for x.

$x = 3$

❑ **Simultaneous Equations in Word Problems** – Word problems that require you to define two variables are often simultaneous equation questions. Learn to recognize them and translate to set up the equations.

1000 tickets were sold to the Seaport Aquarium's Dolphin Show. Adult tickets cost $10, children's tickets cost $2, and a total of $5200 was collected. How many adult tickets were sold?

What are your two variables? Define them.

A = number of Adult tickets sold

C = children tickets sold

Write an equation for the total number of tickets: A + C = 1000

Write an equation for the total cost of the tickets: 10A + 2C = 5200

Solve the simultaneous equations for A. A = 400

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PUT IT TOGETHER

1. If $2m + 5n = 6$ and $m + 4n = -6$, what is the value of n?

A. –6
B. –4
C. 0
D. 2
E. 5

$2m + 5n = 6 \rightarrow 2m + 5n = 6$
$-2(m + 4n = -6) \rightarrow -2m - 8n = 12$
$-3n = 18$
$n = -6$

Stack equations, multiply second equation by -2, and add. Make sure to solve for n and don't waste time solving for m first!

2. At a clothing store, 6 shirts and 3 pants cost $201, and 2 shirts and 3 pants cost $117. What is the cost of 1 shirt?

F. $18
G. $20
H. $21
J. $21.50
K. $22.50

$6s + 3p = 201$
$-(2s + 3p = 117)$
$4s = 84$

Write two equations. One for "6 shirts and 3 pants cost $210" and another for "2 shirts and 3 pants cost $117."

3. An airplane at an altitude of 31,000 feet is ascending at a constant rate of 1,000 feet per minute. Another airplane at an altitude of 39,000 feet is descending at a constant rate of 400 feet per minute. After how many minutes, to the nearest tenth of a minute, will the two planes be at the same altitude?

A. 1.3
B. 5.0
C. 5.7
D. 13.3
E. 20.0

$A_1 = 1000t + 31000$
$A_2 = -400t + 39000$

$1000t + 31000 = -400t = 39000$
$1400t = 8000 \rightarrow t \cong 5.71$ minutes

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Evaluation

1-5 per test E M H

- Evaluation questions ask you to consider an experiment's overall results. Usually, these questions rely on your understanding of the relationships between variables.

- Evaluation questions typically appear in the following forms:

  A biologist claimed that _____. Are the results from Study 1 consistent with her claim?

  It was hypothesized that _____. Is this consistent with the results of Experiment 1?

  Which of the following statements about _____ is supported by the results of Study 2?

- These questions may ask if the experimenter's original hypothesis was verified or not. The answers typically include two "Yes" options and two "No" options, each with different justifications.

Students in a chemistry class wanted to compare the heat energy that could be produced from peanuts, cashews, and almonds. In separate trials, a gram of each substance was burned in an insulated environment, where the energy heated a 100mL beaker of water. The students measured the maximum change in the water temperature while the samples were burnt. All trials were conducted in a room with a stable temperature of 20ºC, and the water began at 20ºC for each trial. In order to calculate the energy (in joules) produced by burning each sample, students used the following formula:

joules = mass of the water × temperature increase × 4.2 J/(gºC)

	temperature increase of water (°C)	energy (joules)
almond	+ 14.9	6276
cashew	+ 19.9	8368
peanut	+ 9.96	4184

Which of the following statements are supported by the results of this study?

A. Almonds and cashews have less oil than peanuts.
B. Almonds produced two times the energy that peanuts produced.
C. One gram of almonds produced less energy than two grams of peanuts.
D. Cashews increased the water temperature by the least amount.

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A student hypothesized that burning the cashew would heat the water more quickly than would burning the peanut or almond. Is this hypothesis consistent with the results of the experiment?

F. Yes, because the water reached the highest temperature.
G. Yes, because the cashew released the most energy when burned.
H. No, because the cashew heated the water more slowly than the almond.
J. No, because time was not a variable measured in this experiment.

- ❑ These questions may introduce new information about the experimental design and ask how the new information affects the results.

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PUT IT TOGETHER

Over a period of 20 years, researchers studied the water quality and animal populations of a large lake ecosystem in Connecticut. Prior to the beginning of the study, the lake had a stable pH of 6.5.

Study 1

Researchers conducted a study to determine the *tolerance* of animal species in the lake environment for various pH ranges in captivity. The researchers began by establishing average birthrates and life expectancies for each species in aqueous environments with a pH of 6.5, as a control value. Later, researchers conducted similar experiments, varying the pH of the environment. The birthrates and life spans of each animal type were averaged. A species with data within 80% of both the control birthrate and life expectancy was deemed "tolerant" to a given pH environment. The results of this study are shown in Table 1.

Table 1

	pH tolerance					
	6.5	6.0	5.5	5.0	4.5	4.0
trout	✓	✓	✓	✓		
bass	✓	✓				
perch	✓	✓	✓	✓	✓	
frogs	✓	✓	✓	✓	✓	✓
salamanders	✓					

5

Study 2

Researchers tested the pH levels in the lake on the first day of each month for the entire 20-year study. Decreases of more than 1 pH per year were attributed to manmade pollution, while fluctuations less than this likely occurred naturally. The data from this study was used to create Figure 1.

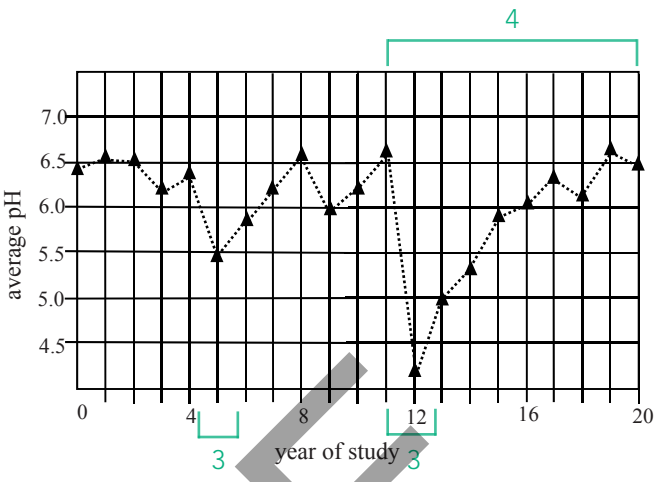

Figure 1

Study 3

Researchers monitored the populations of two fish, trout and bass, over the course of the study. Every 3 months, researchers would spend 8 hours catching, tagging, and releasing fish within the lake. The data is shown in Figure 2 below.

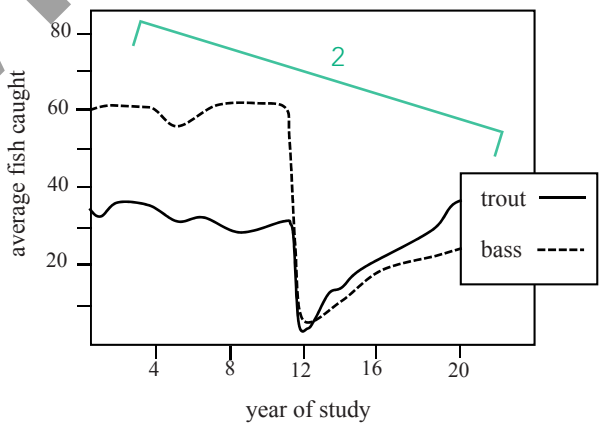

Figure 2

5

How does the data in Figure 1 relate to the date in Figure 2? What would explain the similarities?

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1. A biologist stated that bass would be the only animals in the lake negatively affected if the pH dropped to 5.0. Is this consistent with the data in Table 1?

   A. Yes, because bass are constantly immersed in water.
   B. Yes, because all other animals are tolerant of this pH.
   C. No, because salamanders are not tolerant of this pH.
   D. No, because both salamanders and trout are not tolerant of this pH.

2. An ecology student hypothesized that, over the course of the study, there would always be a higher population of bass than trout in the lake. Does Figure 2 support this claim?

   F. Yes, because there were always more bass caught.
   G. Yes, because bass were more tolerant at the various pH levels.
   H. No, because there are always more trout caught.
   J. No, because trout surpassed bass soon after year 12.

3. An environmental group stated that, during the course of the study, two factories opened in the vicinity of the lake. Both operated for one calendar year before they were forced to shut down due to their negative effects upon the pH of the lake. According to Study 2, which of the following could be the years that these factories opened?

   A. Year 3 and Year 8
   B. Year 4 and Year 8
   C. Year 8 and Year 11
   D. Year 4 and Year 11

4. An ecologist claimed that, when a body of water drops more than 1.5 pH units over the course of a year, it takes longer for it to neutralize its pH than it does for the water to become acidic. Are the results of Study 2 consistent with this claim?

   F. Yes, because it took less time for the pH to return to 6.5.
   G. Yes, because it took more time for the pH to return to 6.5.
   H. No, because it took the same amount of time.
   J. No, because it cannot be determined from the data.

5. Which of the following statements about fish populations is supported by both Table 1 and Figure 2?

   A. Bass prey upon frogs.
   B. Perch populations recovered more quickly than bass from the most drastic pH change.
   C. Decreased acidity harms bass and trout populations.
   D. Increased acidity harms bass and trout populations.

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