The Redesigned SAT & PSAT Course Book

Math

SUMMIT
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Focusing on the Individual Student
The Redesigned SAT & PSAT Course Book

Math

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Focusing on the Individual Student
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The SAT Math Test

- There are 58 math questions that are split between a 25-minute no-calculator section and a 55-minute calculator section.

| Format | 58 questions  
Multiple-choice and grid-ins  
2 sections  
  - 1 with calculator NOT allowed (20 questions)  
  - 1 with calculator allowed (38 questions) |
|--------|--------------------------------------------------|
| Content | Problem Solving and Data Analysis  
Heart of Algebra  
Passport to Advanced Math  
Additional Topics in Math |
| Scoring | Math score: 200-800 |
| Time | 80 minutes  
  - 25 minutes for no-calculator section  
  - 55 minutes for calculator section |

- Each Math Test contains a specific number of questions in each of four content areas:

<table>
<thead>
<tr>
<th>Content Area</th>
<th>Questions</th>
<th>Sample Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving and Data Analysis</td>
<td>19</td>
<td>percents, proportions, statistics</td>
</tr>
<tr>
<td>Heart of Algebra</td>
<td>17</td>
<td>algebraic equations, systems of equations, graphs of linear equations, linear models</td>
</tr>
<tr>
<td>Passport to Advanced Math</td>
<td>16</td>
<td>functions, quadratic equations, polynomials</td>
</tr>
<tr>
<td>Additional Topics in Math</td>
<td>6</td>
<td>angles, triangles, circles, trigonometry, complex numbers</td>
</tr>
</tbody>
</table>

- Problem Solving and Data Analysis questions appear only in the 55-minute calculator-allowed section.
The PSAT Math Test

- There are 48 math questions that are split between a 25-minute no-calculator section and a 45-minute calculator section.

| Format   | 48 questions  
|          | Multiple-choice and Grid-ins  
|          | 2 sections  
|          | - 1 with calculator not allowed (17 questions)  
|          | - 1 with calculator allowed (31 questions)  
| Content  | Problem Solving and Data Analysis  
|          | Heart of Algebra  
|          | Passport to Advanced Math  
|          | Additional Topics in Math  
| Scoring  | Math score: 160-760  
| Time     | 70 minutes  
|          | - 25 minutes for no-calculator section  
|          | - 45 minutes for calculator section  

- The PSAT and SAT are scored on vertically aligned scales. This means, for instance, that a student scoring 550 on the PSAT is demonstrating the same level of achievement as a student scoring 550 on the SAT. It does not mean that the same student is predicted to score a 550 on the SAT. The PSAT is reported on a slightly lower scale (160-760), reflecting the fact that the exams test the same body of skills, but at age-appropriate levels.
# Question Difficulty

Problems progress in rough order of difficulty. You should always know whether you are working on an easy problem, a medium problem, or a difficult problem.

## SAT Math Sections

### 25 Minutes – Calculator NOT Allowed

<table>
<thead>
<tr>
<th>MULTIPLE-CHOICE</th>
<th>EASY</th>
<th>MEDIUM</th>
<th>DIFFICULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15</td>
<td>E</td>
<td>M</td>
<td>D</td>
</tr>
</tbody>
</table>

| GRID-INS | 16 17 18 19 20 | E | M | D |

### 55 Minutes – Calculator Allowed

<table>
<thead>
<tr>
<th>MULTIPLE-CHOICE</th>
<th>EASY</th>
<th>MEDIUM</th>
<th>DIFFICULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30</td>
<td>E</td>
<td>M</td>
<td>D</td>
</tr>
</tbody>
</table>

| GRID-INS | 31 32 33 34 35 36 37 38 | E | M | D |

## PSAT Math Sections

### 25 Minutes – Calculator NOT Allowed

<table>
<thead>
<tr>
<th>MULTIPLE-CHOICE</th>
<th>EASY</th>
<th>MEDIUM</th>
<th>DIFFICULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13</td>
<td>E</td>
<td>M</td>
<td>D</td>
</tr>
</tbody>
</table>

| GRID-INS | 14 15 16 17 | E | M | D |

### 45 Minutes – Calculator Allowed

<table>
<thead>
<tr>
<th>MULTIPLE-CHOICE</th>
<th>EASY</th>
<th>MEDIUM</th>
<th>DIFFICULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27</td>
<td>E</td>
<td>M</td>
<td>D</td>
</tr>
</tbody>
</table>

| GRID-INS | 28 29 30 31 | E | M | D |

Note that the easy-to-difficult structure starts again with grid-ins. So, for instance, #16 will be much easier than #15. For some students, it is best to do the easier grid-ins before attempting the difficult multiple choice questions.
Use the structure of the test to your advantage. Each question is worth 1 raw point. Regardless of its difficulty, every question counts the same. Would you rather earn $20 working for 1 hour or earn $20 working for 10 minutes? The answer is easy, right? Similarly, on the Math test, you should spend your time earning points as efficiently as you can.

Follow these tips:

- Put your time and energy into the questions within your capabilities, starting with the easiest and finishing with the hardest.

- Don’t spend time on a hard question when there are easier questions still available. That’s like passing on an opportunity to earn $20 in 10 minutes!

- When you’re working on harder questions, make sure you check your answers! Suspect an answer that comes a little too easily.

- In each section, consider doing the first few grid-ins prior to doing the last few multiple-choice questions. But don’t forget to go back to the multiple-choice questions you skipped!

The questions in the calculator section are generally more complex than the questions in the no-calculator section. For some questions in the calculator section, your calculator can be a great help; however, some questions can be solved more quickly without a calculator.
Plugging In

As you progress through your preparation, you want to build your arsenal of SAT skills and strategies. Plugging In and Choosing Numbers are two of the most useful math strategies, providing you with a way to make abstract algebra questions more concrete and accessible, and allowing you to solve some very difficult questions.

- Don’t get tunnel vision. If you can’t solve the problem in the forward direction, try to solve it in the reverse direction by plugging in the answer choices.

\[
\begin{align*}
  y &= -2x + 8 \\
  3y &= 5x + 13
\end{align*}
\]

Which ordered pair \((x,y)\) satisfies the system of equations shown above?

A) \((-1,10)\)  
B) \((6,1)\)  
C) \((1,6)\)  
D) \((10,-1)\)

- Plugging In can also be used on difficult word problems.

Mrs. Stone brings some toys to her second-grade class. If each student takes 3 toys, there will be 10 toys left. If 4 students do not take a toy and the rest of the students take 4 toys each, there will be 1 toy left. How many toys did Mrs. Stone bring to the class?

A) 40  
B) 74  
C) 85  
D) 91
Choosing Numbers

- Many SAT Math problems can be solved by choosing your own numbers for variables. We call this strategy Choosing Numbers. Learn to recognize questions that can be solved this way.

- Choosing Numbers is most effective on math problems whose answer choices contain variables, rather than constants. By Choosing Numbers, you'll be able to turn the algebraic expressions into hard numbers. Follow these steps:

1. Choose your own easy numbers to replace the variables. For problems that involve minutes or hours, for instance, you might try 60.

   For problems that involve percents you might try ________.

2. Solve the problem using your numbers.

3. Plug your numbers into all of the answer choices to see which answer choice(s) matches the solution you found in step 2.

4. If your numbers give you two or more correct answers, go back to step 1 and choose different numbers. You do not need to recalculate the choices you have already eliminated.

- Be careful to Choose Numbers that meet any restrictions in the question.
Stay organized by writing down the numbers you choose and the answers you get.

The substance iodine-131 decays at a rate of 50% per 8 days. If a hospital stores 500 grams of iodine-131, which of the following represents the hospital’s remaining stock after \( t \) days?

A) \( 0.5(500)^{\frac{t}{8}} \)

B) \( 8(0.5)^{\frac{500}{t}} \)

C) \( 500(8)^{\frac{t}{5}} \)

D) \( 500(0.5)^{\frac{t}{8}} \)

Choose a number for the number of days, \( t \).

Given that number of days, how many times did the amount of iodine-131 decay by 50%?

Given that number of days, how many grams of iodine-131 would be remaining?

Substitute the number you chose for \( t \) into the answer choices. Which answer choice gives you the answer you arrived at in the previous question?

If a machine can fill \( s \) cartons in one 24-hour day, how many cartons can be filled in \( t \) hours?

A) \( \frac{st}{24} \)

B) \( \frac{24s}{t} \)

C) \( \frac{t}{24s} \)

D) \( 24st \)
Using Your Calculator

You can use a calculator on only one of the two math sections, but even on that section fewer than half of the questions actually require a calculator.

Your calculator can help you compute more efficiently, handle fractions more easily, find points of intersection, and more.

A warning: **your calculator cannot solve problems for you; it is only a tool.** Your calculator is not always the right tool for the job. As you prepare for the SAT, learn to identify when the calculator is most useful and when it will just slow you down. Many questions in the calculator-allowed section do not require a calculator to solve.

- We recommend the TI-83 and TI-84 series of graphing calculators, which are widely used in American high schools. The following instructions are for those series of calculators. These calculators include functionality for fractions, trigonometric functions, and graphing. Although more advanced models exist, some of them are not permitted for use on the SAT.

  Check the College Board website to see which calculator models are allowed for the test.

- Calculators follow strict order of operations. Use parentheses when entering a multi-step calculation, key numbers in carefully, and check the display after each entry.

  Calculate each of the following, first without and then with a calculator.

  Use your calculator to solve:

  \[ -5^2 = \quad \frac{-6 \times 20}{4 \times 5} = \quad \]

- **Working with Fractions** – Use the ▶Frac function to convert decimals or complex fractions into simplified fractions. Press the MATH button; then choose ▶Frac by pressing either 1 or ENTER.

  Use your calculator to solve and put in simplest form:

  \[ \frac{2}{3} \cdot \frac{1}{5} = \quad \frac{52}{455} = \quad \]
Graph the function $f(x) = x^2 - 4$.

Press the \( Y = \) button and enter your equation using the \( X,T,\theta,n \) button for your independent variable.

Press \( \text{GRAPH} \) to see your function in the coordinate plane or \( 2\text{nd} + \text{GRAPH} \) to see a table.

To change the \( x\)- and \( y\)-boundaries of the visible graph, press \( \text{WINDOW} \) and adjust accordingly.

Press \( \text{TRACE} \) and use the arrow keys to follow the coordinates on the line.

Press \( 2\text{nd} \) and \( \text{TRACE} \) to bring up the CALC menu, and select the VALUE feature.

Enter any \( x\)-value to see its position and corresponding \( y\)-value.

What is the value of \( y \) when \( x = 1.5 \)? ________

What are the coordinates of the function’s \( x\)-intercepts? ________  ________

Find the intersections of $f(x) = x^2 - 4$ and $g(x) = x - 2$.

Enter both functions and graph them.

Press \( 2\text{nd} \) and \( \text{TRACE} \) to bring up the CALC menu, and select the INTERSECT feature.

Use the up/down and \( \text{ENTER} \) keys to choose the two functions whose intercept(s) you wish to calculate, and use the left/right keys to select points near the intersection.

Press \( \text{ENTER} \) one more time to calculate the intersection.

How many times do \( f(x) \) and \( g(x) \) intersect? ________

What are the coordinates of the intersection(s)? ____________________

While we can’t list all the circumstances in which your calculator’s advanced functions might be useful, it’s worthwhile to explore its various menus and attempt to use your calculator in creative ways as you practice with SAT Math problems.
Grid-In Questions

13 of 58 math questions are grid-ins – 5 at the end of the no-calculator section and 8 at the end of the calculator section.

- Grid-in questions are just like the multiple-choice questions without the multiple choice answers.

- Grid-in questions progress from easy to difficult.
  Remember that the easy-to-difficult structure starts again with grid-ins. The first grid-in question in a section will be much easier than the last multiple-choice question that comes before it. For some students, it is best to do the easier grid-ins before attempting the difficult multiple-choice questions.

- Although you should always write your answer in the boxes above the words, you receive credit only if the ovals are filled in correctly.

- Start gridding in the far left column to eliminate any indecision.

- Grid-in questions will not have negative numbers as answers.

- **Mixed numbers** must be gridded as improper fractions or decimals.

- **Decimals** must be gridded to the highest degree of accuracy possible.
  In other words, an answer of 0.6666... should be gridded as .666 or .667 or 2/3. .66 or .67 will be marked wrong.

- Some grid-in questions may have more than one correct answer. You need to grid in only one of the correct answers.

- Check your work.
  There are no answer choices to check your answers against, so make sure you check your work before you grid your answer.
Ratios

A ratio compares one quantity to another. On the SAT, ratios are often used for rates (such as miles per hour or salary per year) in real-world word problems.

- **Ratio Basics** – A ratio can be thought of as a comparison between parts of a whole.

  A fruit basket contains 30 oranges and apples. The ratio of apples to oranges is 3 to 2.

  What is the ratio of oranges to apples? ____________

  What fraction of the fruit in the basket is apples? ____________

  How many apples are in the basket? ____________

  If 5 pieces of fruit are randomly picked out of the basket, you would expect to get how many apples and how many oranges? ____________

- **Rates as Ratios** – Ratios are a good way to express rates or some quantity “per” some other quantity. When comparing rates, reduce the fraction so you have 1 in the denominator.

  Which car goes faster? One that travels 150 miles in 5 hours or one that travels 120 miles in 3 hours?

- **Comparing Ratios** – To compare the ratios between multiple pairs of values, write the ratios as fractions and convert to common denominators.

  Thomas has a jar of 60 coins with 16 quarters. Sharon has a jar of 80 coins with 24 quarters. Who has the jar with the largest ratio of quarters to total coins?

- **Probability of an event happening** = \( \frac{\text{# of ways the event can happen}}{\text{# of possible outcomes}} \)

  Probability questions typically involve data tables.
**PUT IT TOGETHER**

Questions 1-2 refer to the following information.

A survey of 374 randomly selected people between age 18 and 41 years old gathered data on whether they have ever been married or not. The results are shown below.

<table>
<thead>
<tr>
<th>Age (18-23)</th>
<th>Married</th>
<th>Never Married</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-23</td>
<td>36</td>
<td>84</td>
<td>120</td>
</tr>
<tr>
<td>24-29</td>
<td>54</td>
<td>36</td>
<td>90</td>
</tr>
<tr>
<td>30-35</td>
<td>79</td>
<td>27</td>
<td>106</td>
</tr>
<tr>
<td>36-41</td>
<td>51</td>
<td>7</td>
<td>58</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>220</td>
<td>154</td>
<td>374</td>
</tr>
</tbody>
</table>

1. Based on the survey results, it is how many times as likely that someone aged 18-23 has married as it is that someone aged 24-29 has never married?
   - A) 4/3
   - B) 3/2
   - C) 1/1
   - D) 3/4

2. Researchers acquired more survey responses. 9 more people aged 36-41 had been married, and a total of 4/5 of the people aged 36-41 had been married. Based on this information, how many of the additional respondents were aged 36-41 and had never married?
   - A) 5
   - B) 8
   - C) 12
   - D) 15

3. Two competing stores offer different sale prices for the same snack item. ShopNow sells the snack in packs of 12 and has a sale offering 3 packs for $10. RitePrice sells the snack in packs of 6 and has a sale offering 4 packs for $7. Which store offers the lower sale price per snack item?
   - A) ShopNow has a lower sale price.
   - B) The sale prices are the same.
   - C) RitePrice has a lower sale price.
   - D) There is not enough information to determine the lower price.
Data Collection and Conclusions

Data collection and conclusions questions ask you to consider how a study is conducted and what you can infer from the study’s results. These questions deal with controlled experiments, observational studies, and sample surveys.

- **Controlled Experiment** – A controlled experiment typically divides subjects into two groups – an experimental group and a control group. No treatment is given to the control group while the experimental group is changed according to some key variable. Otherwise, the two groups are kept under the same conditions.

  A control group is used to create a baseline to which experimental groups can be compared. If an experimental group shows results that a control group does not, it is likely that these results were caused by the variable that the experimenters manipulated.

- **Observational Study** – In an observational study, observations are conducted to monitor changes in variables. Investigators record data and analyze trends without giving any treatment to the variables.

  Observational studies are used to determine the correlation between variables. Keep in mind that an association between two variables does not mean that changes in one variable cause changes in the other.

- **Sample Survey** – In a survey, a sample from a larger population is selected and information from the sample is then generalized to the larger population. The key to the validity of any survey is randomness. Respondents to the survey must be chosen randomly. How well the sample represents the larger population is gauged by two important statistics: **margin of error** and **confidence level**.

  Example: The Pew Research Center conducts a poll (survey) of 1000 voters, and 45% of the respondents say that they will vote for a Candidate Smith. The confidence level is given as 95% and the margin of error is given as 3%. This means that, if the poll were conducted 100 times, the percentage of voters who say that they will vote for Candidate Smith will range between 42% and 48% most (95 percent, actually) of the time.

  Smaller sample sizes have larger margins of error. A sample size of 50, for instance, has a margin of error around 14%, while a sample size of 1000 has a margin of error around 3%. Similarly, the higher the desired confidence level, the larger the sample size needed.
PUT IT TOGETHER

1

A health official conducted a survey of the weights of random adult males in Oklahoma. A sample of 5,000 responses shows that the average weight was 176 pounds. The survey sample has a confidence level of 95% and a margin of error of 8 pounds. Which of the following is the most plausible value for the actual average weight of adult males in Oklahoma?

A) 165 pounds  
B) 182 pounds  
C) 186 pounds  
D) 187 pounds

2

A scientist wants to study the effectiveness of an experimental medication for sinus congestion. The scientist records data on three groups of test subjects. The first group includes people who suffer from sinus congestion and are given the experimental medication. The second group includes people who suffer from sinus congestion and are not given the experimental medication. The third group includes people who do not suffer from sinus congestion and are given the experimental medication. Which of the following best describes the research design for this study?

A) Sample survey  
B) Observational study  
C) Controlled experiment  
D) None of the above

3

A researcher conducting an observational study monitored the age, frequency of mating calls, and mass of male marsh frogs. The researcher observed that weight increased with age (approximately 1 gram per month) and calling rate differed among individual frogs but did not change with age. Based on this data, which conclusion regarding male marsh frogs is valid?

A) There is an association between age and calling rate.  
B) There is no association between weight and calling rate.  
C) An increase in weight causes a decrease in calling rate.  
D) An increase in weight causes an increase in age.
Systems of Linear Equations

Systems of linear equations appear frequently on the SAT. Most often, you are given a pair of equations and then asked to solve. Other times, the two equations are hidden in word problems, requiring you to set up the equations first and then solve. Systems of linear equations can be solved algebraically using the elimination method or substitution method. Some harder questions will present equations that have no solution or infinitely many solutions.

- A system of linear equations, also called simultaneous equations, is a set of two or more equations working together. Simultaneous equations can be solved graphically and algebraically. A system of two linear equations can have no solution, 1 solution, or infinitely many solutions.

- **No Solutions** – When lines are parallel, there is no common solution. What does this mean algebraically? If we consider the slope-intercept forms of the two lines ($y = mx + b$), it means that the slopes ($m$) are the same and the $y$-intercepts ($b$) are different.

- **Infinitely Many Solutions** – When lines overlap, there are infinitely many solutions. Algebraically, this means that the two lines have the same slopes ($m$) and the same $y$-intercepts ($b$).

$$2x + y = 5$$
$$4x + 2y = 10$$

How many solutions does the pair of linear equations shown above have?  

How do you know?
Elimination Method – Add or subtract equations to cancel one of the variables and solve for the other. You may have to multiply an equation by some number to eliminate a variable before the equations are added or subtracted.

If \(2x + y = 3\) and \(-x - 3y = 6\), what is the value of \(y\)?
Stack the equations: 
\[
\begin{align*}
2x + y &= 3 \\
-x - 3y &= 6
\end{align*}
\]
What do you need to multiply the bottom equation by to make the \(x\) disappear when you add the two equations? ____________
Rewrite the equations and add them. Solve for \(y\).

Substitution Method – Solve one equation for one of the variables, and then substitute that value for that variable in the other equation.

If \(2x + y = 3\) and \(-x - 3y = 6\), what is the value of \(x\)?
Solve the equation \(2x + y = 3\) for \(y\): ____________
Substitute that value for \(y\) in the other equation: \(-x - 3(\text{__________}) = 6\)
Solve for \(x\).

Simultaneous Equations in Word Problems – Word problems that require you to define two variables are often simultaneous equation questions. Learn to recognize them and translate to set up the equations.

A high school drama club is raising funds by selling t-shirts and sweaters. Club members sell t-shirts for $12 and sweaters for $25. If the club has sold a total of 16 items for $309, how many t-shirts have been sold?

\(T = \text{number of t-shirts sold}\)
\(S = \text{__________}\)
Write an equation for the total number of items sold: \(T + S = 16\)
Write an equation for the total revenue: ____________
Solve the simultaneous equations for \(T\).
Consider the system of equations above. If \((x, y)\) is the solution to the system, then what is the value of the product of \(x\) and \(y\)?

A) –12
B) –2
C) 1
D) 2

Which ordered pair \((x, y)\) satisfies the system of equations above?

A) (4,15)
B) (12,12)
C) (14,3)
D) (44,15)

The hardcover version of a book sells for $15 and the paperback sells for $11.50. If a store sells 70 copies of the book in one month and charges $917, how many hardcover versions were sold?

Solve both algebraically and by Plugging In answer choices.
In the system of equations above, $a$ and $b$ are constants. If the system has no solutions, what is the value of $\frac{a}{b}$?

A) 12  
B) 3  
C) $\frac{4}{3}$  
D) $\frac{1}{3}$
Creating Linear Models

One typical question type on the SAT is a word problem that describes a real world situation and then asks you to represent that situation algebraically with an expression, equation, or inequality (and sometimes a pair of equations or inequalities).

- Rewrite the definitions of the variables and identify the constants. Rewriting will help clarify the question.

A cosmetics salesperson earns $10 per hour and a 22% commission for his sales. If a salesperson works for 7 hours and sells c dollars’ worth of cosmetics, write an expression for the salesperson’s earned income.

How much did the salesperson earn without the commission? ________

How many dollars’ worth of cosmetics did the salesperson sell? ________

How much did the salesperson earn from these sales? ________

Write an expression for the salesperson’s total earnings: _________________________

- Some linear model questions can be solved by Choosing Numbers.

After summiting the top of a mountain that is 3000 feet in elevation, a hiker descends at a constant rate of 800 feet per hour. Which of the following best describes the hiker’s elevation h hours after she begins her descent from the summit?

What is the hiker’s elevation before she begins her descent, when h = 0? ________

What is the hiker’s elevation after 2 hours, when h = 2? ________

Which of the following answer choices matches this information?

A) \( f (h) = 800h - 3000 \)

B) \( f (h) = 2200h \)

C) \( f (h) = 3000 - h \)

D) \( f (h) = 3000 - 800h \)
PUT IT TOGETHER

1. On a recent fishing trip, Art caught $m$ fish each hour for 4 hours and Brian caught $n$ fish each hour for 4 hours. Which of the following represents the total number of fish Art and Brian caught?
   
   A) $4mn$
   B) $4m + 4n$
   C) $8mn$
   D) $8m + 8n$

2. Lawrence is buying some new tools for his toolbox but only has $\$100$ to spend. After buying three wrenches and two pliers, he still needs two screwdrivers. The wrenches cost $\$12$ each and the pliers $\$14$ each. If $x$ represents the dollar amount he can spend on screwdrivers, which of the following inequalities could be used to determine possible values for $x$?
   
   A) $(3)(12) + (2)(14) - x \leq 100$
   B) $(3)(12) + (2)(14) - x \geq 100$
   C) $(3)(12) + (2)(14) + x \leq 100$
   D) $(3)(12) + (2)(14) + x \geq 100$

3. A doughnut shop has fixed daily costs of $\$750.00$. Its variable costs come from doughnut ingredients, which average $\$0.23$ per doughnut. If doughnuts are sold for $\$0.40$ each, which of the following expressions could be used to predict the profit from producing $d$ doughnuts in a single day?
   
   A) $(0.40 - 0.23)d - 750.00$
   B) $750.00d + (0.40 - 0.23)$
   C) $750.00 - (0.40 + 0.23)d$
   D) $750.00 + (0.40 + 0.23)d$
Graphs of Quadratics

The SAT requires that you understand and can move fluidly among the different forms of quadratic functions: Standard form, Intercept form, and Vertex form. The intercept and vertex forms are useful for graphing quadratic functions. The graph of a quadratic equation is called a parabola.

- **Standard form:** \( y = ax^2 + bx + c \)

- **Intercept form:** \( y = a(x - p)(x - q) \)

  In intercept form, \( p \) and \( q \) are the \( x \)-intercepts, where \( f(x) = 0 \).

  Convert a quadratic equation from standard form to intercept form by factoring.

- **Vertex form:** \( y = a(x - h)^2 + k \)

  In vertex form, \((h,k)\) are the coordinates of the vertex of the parabola.
If a quadratic equation is in standard form, you can convert it to vertex form by “completing the square.”

For an expression \( x^2 + bx \), rewrite as \( \left(x + \frac{b}{2}\right)^2 \), then FOIL and rebalance the equation.

\[ y = 2x^2 + 12x - 8 \]

Step 1: Bring the "loose" term (8) over to the left side.

________________________________

Step 2: Factor out the coefficient on the squared term from both terms on the right.

________________________________

Step 3: For your quadratic expression of the form \( x^2 + bx \), write a new expression of the form \( \left(x + \frac{b}{2}\right)^2 \).

Note that these two expressions are not equal. \( \left(x + \frac{b}{2}\right)^2 = x^2 + bx + \frac{b^2}{4} \).

The new expression is exactly \( \frac{b^2}{4} \) more than the original. To compensate for this, you must add the product of \( \frac{b^2}{4} \) and its coefficient to the left side. This keeps the equation balanced.

________________________________

Step 4: Rewrite the equation in vertex form.

________________________________

Step 5: What are the coordinates of the parabola’s vertex?

________________________________
Put It Together

\[ y = 2x(x + 1) - 6(x + 1) \]

Which of the following is the graph in the \(xy\)-plane of the equation shown above?

A) [Graph A]

B) [Graph B]

C) [Graph C]

D) [Graph D]

Which graph has \(x\)-intercepts that are zeros of the equation?
The graph of \( y = f(x) \) in the \( xy \)-plane is a parabola with vertex at \((2, -27)\), as shown above. Which of the following is an equivalent form of the equation which shows the \( x \)-intercepts of the parabola as constants?

A) \( f(x) = 3(x - 5)(x - (-1)) \)

B) \( f(x) = 3(x - 11)^2 \)

C) \( f(x) = (3x - 9)(x - 2) \)

D) \( f(x) = 3x^2 - 12x - 15 \)

The graph of the equation above is a parabola in the \( xy \)-plane. In which of the following equivalent forms of the function \( f \) do the \( xy \)-coordinates of the vertex of the parabola appear as constants or coefficients?

A) \( f(x) = 2(x - 1)(x + 3) \)

The \( x \)-intercepts of a parabola appear as constants when a quadratic equation is written in intercept form.
### Equations with Fractions

#### Question 1:
If \( \frac{12x}{y} = 2 \), what is the value of \( \frac{y}{x} \)?

A) 2  
B) 4  
C) 6  
D) 7

#### Question 2:

\[
\frac{2}{(x+2)^2 - 6(x+2) + 9}
\]

For what value of \( x \) is the above expression undefined?

#### Question 3:
If \( j \) is an integer, and \( \frac{3j-2}{j} = 3j - 2 \), which of the following best describes the solution set to the equation shown above?

A) The equation has exactly one solution, \( j = 1 \).  
B) The equation has exactly one solution, \( j = 0 \).  
C) The equation has no solutions.  
D) The equation has infinitely many solutions.

### Equations with Exponents

#### Question 4:
If \( x^3 = a \) and \( x^7 = b \), which of the following must be equal to \( x^8 \)?

A) \( a^2 b^2 \)  
B) \( b^2 \)  
C) \( a b^2 \)  
D) \( 2(b - a) \)

#### Question 5:

\( 6m^2 n^7 \) is the product of \( 3mn \) and

A) \( 2m^2 n^2 \)  
B) \( 2m^2 n \)  
C) \( 2m^2 n^3 \)  
D) \( 2mn^2 \)

#### Question 6:
If \( 3^v = 9^2 \), then \( n = \)

\[
1^1 \quad 2^2 \quad 3^3 \quad 4^4 \quad 5^5 \quad 6^6 \quad 7^7 \quad 8^8 \quad 9^9
\]
Radical Equations

Question 7: E
Question 8: M

7

\[
\left( \sqrt{\frac{1}{3}} + \frac{1}{6} \right) \sqrt{2} = \]
A) \( \frac{\sqrt{2}}{2} \)
B) \( 1 \)
C) \( \frac{3}{2} \)
D) \( \frac{3\sqrt{2}}{2} \)

8

If \( \sqrt{3x^2} = 3 \), what is the value of \(|x|\)?

Functions

Question 9: E
Questions 10-11: M

9

If \( f(x) = x^2 - kx - 8 \), and \( f(2) = 0 \), what is the value of \( k \)?
A) \(-4\)
B) \(-2\)
C) \(0\)
D) \(2\)

10

\[ f(x) = \frac{ax^2 + 12}{6 - x} \]

For the function \( f \) defined above, \( a \) is constant and \( f(4) = 10 \). What is the value of \( f(-4) \)?
A) \( \frac{-10}{3} \)
B) \( 2 \)
C) \( \frac{9}{2} \)
D) \( 10 \)

11

Let \( f(x) = ax^2 + bx + c \) for all real numbers \( x \).
If \( f(0) = 2 \) and \( f(1) = 1 \), then \( a + b = \)
A) \(-2\)
B) \(-1\)
C) \(1\)
D) \(2\)
Volume

The test instructions for the Math Test include virtually all of the important geometry formulas, including those for volume. Most volume questions on the test involve cylinders and cones, such as calculating the volume of a storage silo.

The following volume formulas are provided in the test instructions. Memorizing them will save you the time of looking back during the test.

\[
V = lwh \quad V = \pi r^2 h \quad V = \frac{4}{3} \pi r^3 \quad V = \frac{1}{3} \pi r^2 h \quad V = \frac{1}{3} lwh
\]

Set up volume questions using the correct formula. Plug in the values you know and solve the resulting algebraic equation for what you don’t know.

Make sure you answer the question that is being asked. For example, if the question is asking for diameter and you’ve found the radius, you’ll have to double it.

Volume questions may require you to use the actual value of \( \pi \) rather than the symbol, \( \pi \). Remember that \( \pi \) is approximately equal to 3.14.

\[23 \pi \text{ cubic meters} = \text{__________ cubic meters}\]
Put It Together

1. A container in the shape of the right circular cone above has a volume of $7290\pi$ cm$^3$. What is the diameter, in centimeters, of the base of the cone?

![Diagram of a cone with a height of 30 cm]

2. A glass décor manufacturer produces right cylindrical vases with internal diameter of 12 cm and a height of 25 cm. If the manufacturer designs a smaller right cylindrical vase that holds 30% as much water and has a height of 30 cm, what is the internal diameter, in centimeters, of the smaller vase?

   A) 36  
   B) 18  
   C) 6   
   D) 3