

The **ACT** Course Book

MATHEMATICS & SCIENCE

SAMPLE

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ISBN: 978-0-692-91426-7

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ANSWER KEY

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Guesstimating

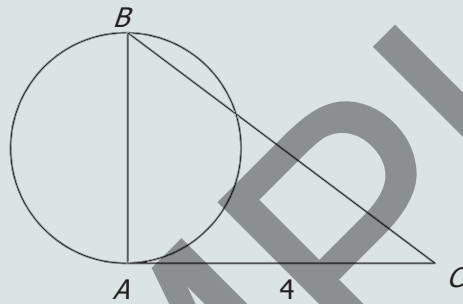
- Most figures on the ACT are drawn to scale. Therefore, a viable strategy for many geometry questions is to use the figure to guide your reasoning or narrow down answer choices.

Note that the ACT states that you should assume “illustrative figures are NOT necessarily drawn to scale.” Usually, figures not drawn to scale will have a disclaimer stating so.

- Guesstimate on figures by comparing unknown values to known values. Use your answer sheet or pencil as a “ruler.”

In the figure below, ~~if \overline{AB} is a diameter of the circle and the circle has radius 1.5~~, what is the length of \overline{BC} ?

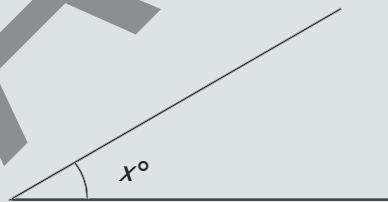
- A. 3
- B. 4
- C. 5
- D. 7
- E. 8



- Draw 90° and 45° angles to help you guesstimate angle measures.

Guesstimate the measure of the angle below.

$x =$ _____



- Know the following approximations to help you guesstimate lengths.

$$\sqrt{2} \approx 1.4$$

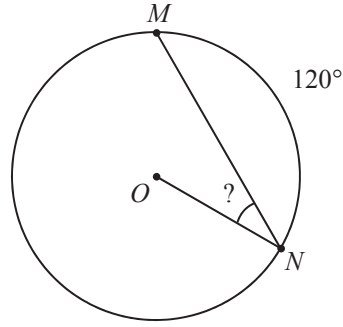
$$\sqrt{3} \approx 1.7$$

$$\pi \approx 3.1$$

PUT IT TOGETHER

Guesstimate to solve the following:

1. In the figure below, the circle has center O and the measure of \widehat{MN} is 120° . What is the measure of $\angle MNO$?



- A. 30°
 B. 40°
 C. 45°
 D. 60°
 E. 80°
2. Of the 398 seniors in a certain high school, approximately 51% are in the art club. Of the seniors who are in the art club, approximately 97% paint with acrylics. Which of the following is closest to the number of seniors who paint with acrylics?
- F. 75
 G. 100
 H. 150
 J. 200
 K. 400

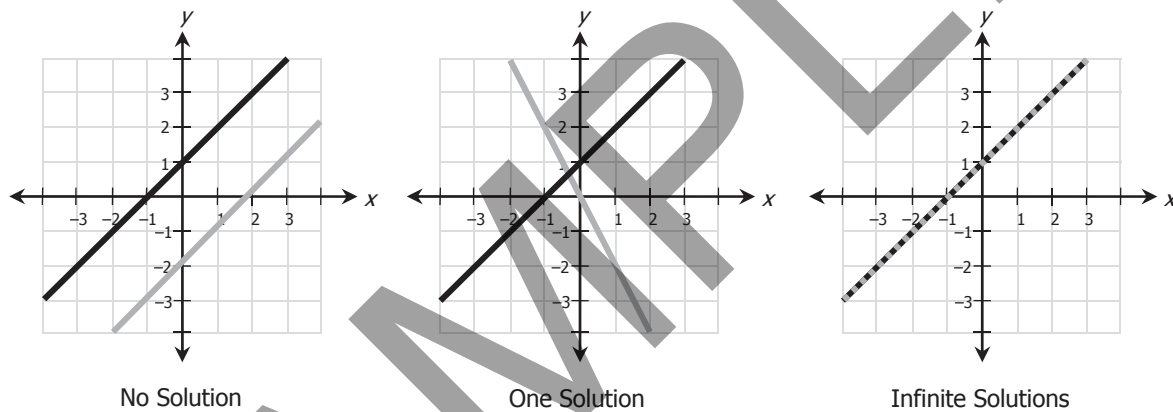
Systems of Equations

0-2 per test E M H

On the ACT, simultaneous equation questions ask you to find a mutual solution for two linear equations. When simultaneous equations are graphed, the mutual solution is the point of intersection.

Some simultaneous equation questions will give the two equations and ask you to solve. Other times, the equations are hidden in a word problem, requiring you to set up the equations first and then solve.

- A system of linear equations, also called simultaneous equations, is a set of two or more equations working together. Simultaneous equations can be solved graphically and algebraically. A system of two linear equations can have no solution, 1 solution, or infinitely many solutions.



- **No Solutions** – When lines are parallel, there is no common solution. What does this mean algebraically? If we consider the slope-intercept forms of the two lines ($y = mx + b$), it means that the slopes (m) are the same and the y -intercepts (b) are different.
- **Infinitely Many Solutions** – When lines overlap, there are infinitely many solutions. Algebraically, this means that the two lines have the same slopes (m) and the same y -intercepts (b).

$$2x + y = 5$$

$$4x + 2y = 10$$

How many solutions does the pair of linear equations shown above have? _____

How do you know?

- ❑ **Elimination Method** – Add or subtract equations to cancel one of the variables and solve for the other. You may have to multiply an equation by some number to eliminate a variable before the equations are added or subtracted.

If $2x + y = 16$ and $x - 2y = 3$, what is the value of x ?

Stack the equations: $2x + y = 16$
 $x - 2y = 3$

What do you need to multiply the top equation by to make the y disappear when you add the two equations? _____

Rewrite the equations and add them. Solve for x .

- ❑ **Substitution Method** – Solve one equation for one of the variables, and then substitute that value for that variable in the other equation.

If $2x + y = 3$ and $-x - 3y = 6$, what is the value of x ?

Solve the equation $2x + y = 3$ for y : _____

Substitute that value for y in the other equation: $-x - 3(\text{_____}) = 6$

Solve for x .

- ❑ **Simultaneous Equations in Word Problems** – Word problems that require you to define two variables are often simultaneous equation questions. Learn to recognize them and translate to set up the equations.

1000 tickets were sold to the Seaport Aquarium's Dolphin Show. Adult tickets cost \$10, children's tickets cost \$2, and a total of \$5200 was collected. How many adult tickets were sold?

What are your two variables? Define them.

A = number of Adult tickets sold

C = _____

Write an equation for the total number of tickets: $A + C = 1000$

Write an equation for the total cost of the tickets: _____

Solve the simultaneous equations for A.

PUT IT TOGETHER

1. If $2m + 5n = 6$ and $m + 4n = -6$, what is the value of n ?
 - A. -6
 - B. -4
 - C. 0
 - D. 2
 - E. 5

2. At a clothing store, 6 shirts and 3 pants cost \$201, and 2 shirts and 3 pants cost \$117. What is the cost of 1 shirt?
 - F. \$18
 - G. \$20
 - H. \$21
 - J. \$21.50
 - K. \$22.50

3. An airplane at an altitude of 31,000 feet is ascending at a constant rate of 1,000 feet per minute. Another airplane at an altitude of 39,000 feet is descending at a constant rate of 400 feet per minute. After how many minutes, to the nearest tenth of a minute, will the two planes be at the same altitude?
 - A. 1.3
 - B. 5.0
 - C. 5.7
 - D. 13.3
 - E. 20.0

Write two equations. One for "6 shirts and 3 pants cost \$210" and another for "2 shirts and 3 pants cost \$117."

Evaluation

1-5 per test E M H

- Evaluation questions ask you to consider an experiment's overall results. Usually, these questions rely on your understanding of the relationships between variables.

- Evaluation questions typically appear in the following forms:

A biologist claimed that _____. Are the results from Study 1 consistent with her claim?

It was hypothesized that _____. Is this consistent with the results of Experiment 1?

Which of the following statements about _____ is supported by the results of Study 2?

- These questions may ask if the experimenter's original hypothesis was verified or not. The answers typically include two "Yes" options and two "No" options, each with different justifications.

Students in a chemistry class wanted to compare the heat energy that could be produced from peanuts, cashews, and almonds. In separate trials, a gram of each substance was burned in an insulated environment, where the energy heated a 100mL beaker of water. The students measured the maximum change in the water temperature while the samples were burnt. All trials were conducted in a room with a stable temperature of 20°C, and the water began at 20°C for each trial. In order to calculate the energy (in joules) produced by burning each sample, students used the following formula:

$$\text{joules} = \text{mass of the water} \times \text{temperature increase} \times 4.2 \text{ J}/(\text{g}^\circ\text{C})$$

	temperature increase of water (°C)	energy (joules)
almond	+ 14.9	6276
cashew	+ 19.9	8368
peanut	+ 9.96	4184

Which of the following statements are supported by the results of this study?

- Almonds and cashews have less oil than peanuts.
- Almonds produced two times the energy that peanuts produced.
- One gram of almonds produced less energy than two grams of peanuts.
- Cashews increased the water temperature by the least amount.

A student hypothesized that burning the cashew would heat the water more quickly than would burning the peanut or almond. Is this hypothesis consistent with the results of the experiment?

- F. Yes, because the water reached the highest temperature.
- G. Yes, because the cashew released the most energy when burned.
- H. No, because the cashew heated the water more slowly than the almond.
- J. No, because time was not a variable measured in this experiment.

- These questions may introduce new information about the experimental design and ask how the new information affects the results.

SAMPLE

PUT IT TOGETHER

Over a period of 20 years, researchers studied the water quality and animal populations of a large lake ecosystem in Connecticut. Prior to the beginning of the study, the lake had a stable pH of 6.5.

Study 1

Researchers conducted a study to determine the *tolerance* of animal species in the lake environment for various pH ranges in captivity. The researchers began by establishing average birthrates and life expectancies for each species in aqueous environments with a pH of 6.5, as a control value. Later, researchers conducted similar experiments, varying the pH of the environment. The birthrates and life spans of each animal type were averaged. A species with data within 80% of both the control birthrate and life expectancy was deemed “tolerant” to a given pH environment. The results of this study are shown in Table 1.

Table 1						
	pH tolerance					
	6.5	6.0	5.5	5.0	4.5	4.0
trout	✓	✓	✓	✓		
bass	✓	✓				
perch	✓	✓	✓	✓	✓	
frogs	✓	✓	✓	✓	✓	✓
salamanders	✓					

Study 2

Researchers tested the pH levels in the lake on the first day of each month for the entire 20-year study. Decreases of more than 1 pH per year were attributed to manmade pollution, while fluctuations less than this likely occurred naturally. The data from this study was used to create Figure 1.

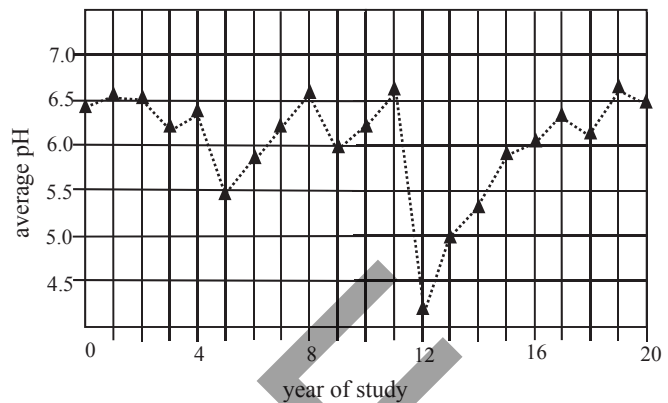


Figure 1

Study 3

Researchers monitored the populations of two fish, trout and bass, over the course of the study. Every 3 months, researchers would spend 8 hours catching, tagging, and releasing fish within the lake. The data is shown in Figure 2 below.

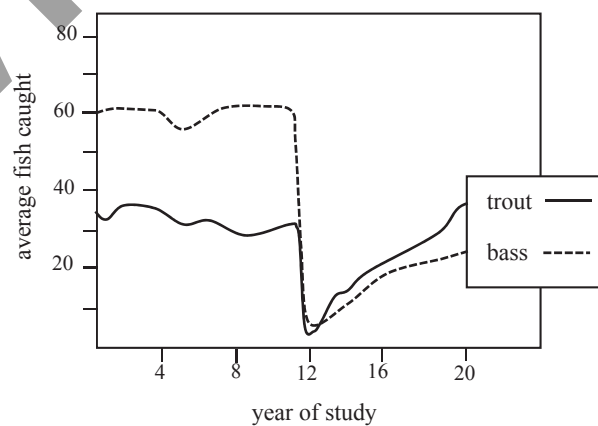


Figure 2

How does the data in Figure 1 relate to the data in Figure 2? What would explain the similarities?

1. A biologist stated that bass would be the only animals in the lake negatively affected if the pH dropped to 5.0. Is this consistent with the data in Table 1?
 - A. Yes, because bass are constantly immersed in water.
 - B. Yes, because all other animals are tolerant of this pH.
 - C. No, because salamanders are not tolerant of this pH.
 - D. No, because both salamanders and trout are not tolerant of this pH.

2. An ecology student hypothesized that, over the course of the study, there would always be a higher population of bass than trout in the lake. Does Figure 2 support this claim?
 - F. Yes, because there were always more bass caught.
 - G. Yes, because bass were more tolerant at the various pH levels.
 - H. No, because there are always more trout caught.
 - J. No, because trout surpassed bass soon after year 12.

3. An environmental group stated that, during the course of the study, two factories opened in the vicinity of the lake. Both operated for one calendar year before they were forced to shut down due to their negative effects upon the pH of the lake. According to Study 2, which of the following could be the years that these factories opened?
 - A. Year 3 and Year 8
 - B. Year 4 and Year 8
 - C. Year 8 and Year 11
 - D. Year 4 and Year 11

4. An ecologist claimed that, when a body of water drops more than 1.5 pH units over the course of a year, it takes longer for it to neutralize its pH than it does for the water to become acidic. Are the results of Study 2 consistent with this claim?
 - F. Yes, because it took less time for the pH to return to 6.5.
 - G. Yes, because it took more time for the pH to return to 6.5.
 - H. No, because it took the same amount of time.
 - J. No, because it cannot be determined from the data.

5. Which of the following statements about fish populations is supported by both Table 1 and Figure 2?
 - A. Bass prey upon frogs.
 - B. Perch populations recovered more quickly than bass from the most drastic pH change.
 - C. Decreased acidity harms bass and trout populations.
 - D. Increased acidity harms bass and trout populations.