

The **SAT** & PSAT Course Book

Math

Instructor's Edition

SAMPLE



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Focusing on the Individual Student

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ANSWER KEY

184

Explanation of Instructor's Edition Notes:

- Throughout this book, helpful hints are included in green. Students will not have these notes in their version of the textbook.
- Answers are located at the bottom of each practice page.
- Selected attractor answer choices are denoted by the symbol .
- Additional questions are indicated by the symbol .

These questions can improve understanding or expand skills and are not pertinent to solving the main problem.

Problem-Solving Tools

You need a set of several strategies for solving different types of SAT Math problems. Learn to adapt and try different strategies when you get stuck on a problem.

❑ **Write in your test booklet.**

You have space in the test booklet for a reason. Use it.

Be a proactive test-taker. Extract key pieces of information and write them down; write down the solution steps; draw a graph; make a table; label geometric figures; cross out incorrect answers. Practice this skill throughout your test preparation. You'll find yourself making fewer careless mistakes and clarifying solutions.

❑ **Don't erase.**

Don't waste time erasing calculations that you've mangled; just put a slash through them. It's faster.

❑ **Irrelevant information**

On rare occasions, an SAT math question will contain information that is not required to solve the question. If you've solved a question without using all of the information in the problem, it's very possible that you've done everything right.

❑ **Use process of elimination (POE).**

If you can't get to the right answer in a straightforward manner, look to eliminate answer choices. Consider the values and situation in the question, and eliminate answer choices that cannot logically work. The more answer choices you can eliminate, the greater advantage you have.

❑ **Check your work, quickly.**

Do a quick check after you do each question. Don't wait until the end of the section to check your work.

The test writers predict potential mistakes by students and include those mistakes as answer choices. These answer choices are considered "attractors" because they seem correct if you do not fully understand or fully solve the question.

Ask yourself:

- Did I find the number the question is asking for? Did you find x , but the question asks for $2x+4$? Did you find the radius, but the question asks for the diameter? Don't celebrate too soon!
- Can I quickly verify my answer? Can I use my calculator to verify? Can I plug in an answer choice to a given equation?
- Is my answer reasonable and logical given the context of the question?

For the following problem, determine how a student might arrive at each of the incorrect answer choices:

$$x - 2y = 2$$

$$2x + y = 9$$

For the system of equations shown above, what is the value of $x - y$?

- ⚠ A) -3
- ⚠ B) 1
- C) 3
- ⚠ D) 4

How can you check your answer?

Plug in values for x and y into the original equations.

Why are 1 and 4 included as answer choices?

Those are the values of x and y , but the question is asking for the difference of x and y .

Why is -3 included as an answer choice?

-3 is equal to $y - x$. If you mix up the x and y values, you will mistakenly get this answer.

Plugging In

As you progress through your preparation, you want to build your arsenal of SAT skills and strategies. Plugging In and Choosing Numbers are two of the most useful math strategies, providing you with a way to make abstract algebra questions more concrete and accessible, and allowing you to solve some very difficult questions.

- ❑ Don't get tunnel vision. If you can't solve the problem in the forward direction, try to solve it in the reverse direction by plugging in the answer choices.

$$y = -2x + 8$$

$$3y = 5x + 13$$

Which ordered pair (x,y) satisfies the system of equations shown above?

- A) $(-1,10)$
- B) $(6,1)$
- C) $(1,6)$
- D) $(10,-1)$

C

Test both equations!

Note that choice A $(-1,10)$ works in the first equation, but in the second equation it results in the impossible statement $30 = 8$, making it incorrect.

- ❑ Plugging In can also be used on difficult word problems.

Mrs. Stone brings some toys to her second-grade class. If each student takes 3 toys, there will be 10 toys left. If 4 students do not take a toy and the rest of the students take 4 toys each, there will be 1 toy left. How many toys did Mrs. Stone bring to the class?

- A) 40
- B) 74
- C) 85
- D) 91

C

In each case, subtract the remainder from the answer choices.
After subtracting 10, is the answer divisible by 3?
After subtracting 1, is the answer divisible by 4?

To solve algebraically, translate each statement into an equation, then set them equal to each other.
 $t = 3x + 10$ and $t = 4(x - 4) + 1$.

In some cases, you can look at the answer choices to see what form of answer is needed. This can help guide your way to the solution.

Choosing Numbers

- ❑ Many SAT Math problems can be solved by choosing your own numbers for variables. We call this strategy Choosing Numbers. Learn to recognize questions that can be solved this way.

- ❑ Choosing Numbers is most effective on math problems whose answer choices contain variables, rather than constants. By Choosing Numbers, you'll be able to turn the algebraic expressions into hard numbers. Follow these steps:
 1. Choose your own easy numbers to replace the variables. For problems that involve minutes or hours, for instance, you might try 60.

For problems that involve percents you might try 100.
 2. Solve the problem using your numbers.
 3. Plug your numbers into **all** of the answer choices to see which answer choice(s) matches the solution you found in step 2.
 4. If your numbers give you two or more correct answers, go back to step 1 and choose different numbers. You do not need to recalculate the choices you have already eliminated.

- ❑ Be careful to Choose Numbers that meet any restrictions in the question.

- Stay organized by writing down the numbers you choose and the answers you get.

The substance iodine-131 decays at a rate of 50% per 8 days. If a hospital stores 500 grams of iodine-131, which of the following represents the hospital's remaining stock after t days?

- A) $0.5(500)^{\frac{t}{8}}$
 B) $8(0.5)^{\frac{t}{500}}$
 C) $500(8)^{\frac{t}{0.5}}$
 D) $500(0.5)^{\frac{t}{8}}$

When choosing numbers, use values that will work well with the situation in the question.

Given a rate of 50% per 8 days, what numbers might you choose for t ?

Choose a number for the number of days, t .

Try starting with 16.

Given that number of days, how many times did the amount of iodine-131 decay by 50%?

The substance will have decayed by 50% two times. We know this from the problem, which states that the substance decays by 50% per 8 days.

Given that number of days, how many grams of iodine-131 would be remaining?

The first 8 days would bring the sample to 250 grams, then the second 8 days would give a final value of 125 grams.

Substitute the number you chose for t into the answer choices. Which answer choice gives you the answer you arrived at in the previous question?

D

If a machine can fill s cartons in one 24-hour day, how many cartons can be filled in t hours?

- A) $\frac{st}{24}$ A Choose 24 for s . This would mean the machine can fill one carton per hour.
 Choose 3 for t (hours the machine is running).
- B) $\frac{24s}{t}$ Logically, we know that if the machine is working at this rate, it will fill 3 cartons in 3 hours.
- C) $\frac{t}{24s}$ If we plug these values into the equations, only choice A will result in a value of 3.
- D) $24st$

Using Your Calculator

You can use a calculator on only one of the two math sections, but even on that section fewer than half of the questions actually require a calculator.

Your calculator can help you compute more efficiently, handle fractions more easily, find points of intersection, and more.

A warning: **your calculator cannot solve problems for you; it is only a tool.** Your calculator is not always the right tool for the job. As you prepare for the SAT, learn to identify when the calculator is most useful and when it will just slow you down. Many questions in the calculator-allowed section do not require a calculator to solve.

- ❑ We recommend the TI-83 and TI-84 series of graphing calculators, which are widely used in American high schools. The following instructions are for those series of calculators. These calculators include functionality for fractions, trigonometric functions, and graphing. Although more advanced models exist, some of them are not permitted for use on the SAT.

Check the College Board website to see which calculator models are allowed for the test. [Make sure to practice with the same calculator that you will be using on the test!](#)

- ❑ Calculators follow strict order of operations. Use parentheses when entering a multi-step calculation, key numbers in carefully, and check the display after each entry.

Calculate each of the following, first without and then with a calculator.

Use your calculator to solve:

$$-5^2 = \underline{-25} \qquad (-5)^2 = \underline{25} \qquad \frac{-6 \times 20}{4 \times 5} = \underline{-6}$$

- ❑ **Working with Fractions** – Use the ►Frac function to convert decimals or complex fractions into simplified fractions. Press the **MATH** button; then choose ►Frac by pressing either 1 or **ENTER**.

Use your calculator to solve and put in simplest form:

$$\frac{2}{3} - \frac{1}{5} = \underline{\frac{7}{15}} \qquad \frac{52}{455} = \underline{\frac{4}{35}}$$

In your sessions, guide students as to when they should (or should not) reach for the calculator. Encourage the use of logic and build arithmetic skills by doing problems by hand.

Percents – Part 2 1-4 per test

- ❑ **Percent Increase/Decrease** – To find the percent increase or decrease from one number to another, divide the difference between the numbers by the original number, then convert the resulting decimal to a percent.

The price of a printer is marked down from \$200 to \$150. What is the percent markdown in price? **25%**
(This can be done easily without a calculator)

- ❑ **Increasing (or Decreasing) by a Percent** – To change a number by a percent, find the percent of the number and then add it to (or subtract it from) the original number. You can also multiply the original number by 100% plus or minus the percent change.

In 1993, the population of Butterdale increased by 16%. If the population was 12,025 at the beginning of the year, what was the population by the end of the year?
 $12,025(1.16) = 13,949$

- ❑ Many percent word problems on the SAT require you to set up algebraic expressions or equations as part or all of the solution.

Joshua pays an 18% tip on his lunch bill. Write an algebraic expression that represents Joshua's total bill, including tip.

Choose a variable for Joshua's lunch bill: **B** _____

In terms of the variable, what is Joshua's tip? **$0.18B$** _____

In terms of the variable, what is Joshua's total bill? **$1.18B$** _____

A car repair shop charges a 12% markup on all parts used. If a bill charges a total of \$691.88 for parts, what was the cost of the parts before the markup?

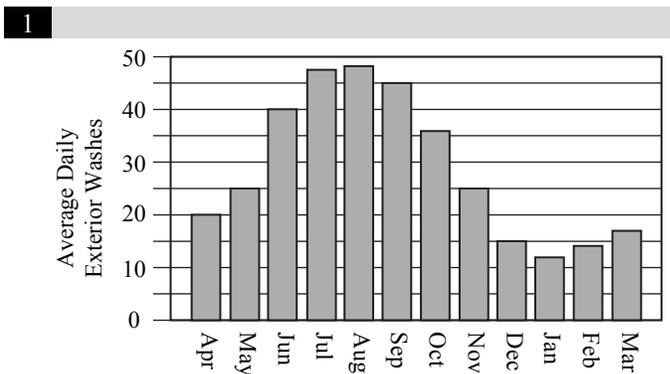
\$617.75

- ❑ **Multiple Percent Changes** – On percent questions that ask you to make two or more percent changes to a number, attack one change at a time. Don't just add or subtract the percents.

A potential car buyer makes an offer for a car that is 80% of the sticker price. The salesperson makes a counteroffer that is 10% higher than the buyer's initial offer. What percent of the sticker price is the salesperson's counteroffer?

88%

PUT IT TOGETHER



The chart above shows the average number of daily transactions at a car wash over the course of one year. Which of the following is closest to the percent increase in average daily exterior washes from May to June?

- A) 15%
- B) 37.5%
- C) 60%
- D) 62.5%

$$\frac{\text{difference}}{\text{original \#}} \times 100$$

2 Students in a science class determine that a can of regular soda has an average of 5% more weight than a can of diet soda. Based on this information, if 24 cans of regular soda weigh 9576 grams, which of the following is the weight, to the nearest gram, of one can of diet soda?

- A) 419 grams *student multiplied by 1.05 instead of dividing.*
- B) 399 grams *this is the weight of one can of regular soda.*
- C) 394 grams
- D) 380 grams

Use your calculator to work quickly and avoid careless mistakes on calculations involving large numbers.

$$9576 / 24 = 399 \text{ grams per regular soda}$$

$$399 / 1.05 = 380 \text{ grams per diet soda}$$

3 A store increased the price of a computer by 10% and then discounted the computer by 30%. If the original price of the computer was p , and the price after the discount was c , what is the relationship between c and p ?

- A) $c = 0.2p$
- B) $c = (1.1)(0.7)p$
- C) $c = (0.1)(0.3)p$
- D) $c = \frac{1.1p}{1.3p}$

If you can't solve algebraically, try Choosing Numbers.

? If the price of a car is discounted by 50%, and then that sale price is discounted by 50%, what percent is the final sale price of the original?

HOMEWORK

- pg 54, #3-4
- pg 58, #21
- pg 59, #23

Algebraic Expressions 2-3 per test

Algebraic expression questions are relatively straightforward, typically requiring some degree of simplifying or factoring.

- ❑ An algebraic expression is an expression that includes one or more variables; it is not an equation. $-2(2x + 3)$ is an algebraic expression.
- ❑ **Simplifying** – To simplify an algebraic expression, expand and combine like terms. To expand, you'll need to know the **Distributive Property** and the **FOIL** method.

Simplify:

$$(k^2 - k + 4 + 2k - 3) - (k + 3k - 4) = \underline{k^2 - 3k - 5}$$

- ❑ **Distributive Property** – When multiplying a single term by an expression inside parentheses, the single term must be multiplied by each term inside the parentheses.

$$-3(2x^2 + x - 3) = \underline{-6x^2 - 3x + 9}$$

- ❑ **F.O.I.L.** – When multiplying two binomials, each term must be multiplied by each term in the other binomial. Use the FOIL method: multiply the first terms, outside terms, inside terms, and last terms.

$$(x - 1)(x + 5) = \underline{x^2 + 4x - 5}$$

- ❑ **Factoring** – Factoring is expanding in reverse. In general, if you see something that can be factored, do it.

Factor:

$$(3x^4 - 12x^3 + 12x^2) = \underline{3x^2(x - 2)^2}$$

Memorize the following quadratics.

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ba + (-b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - ab + ba - b^2 = a^2 - b^2$$

? If $a^2 - b^2 = 40$ and $a - b = 4$, what is the value of b ?

3

? If $(x + 2)(x - 2) = 5$, what is the value of x^2 ?

9

PUT IT TOGETHER



1

$$(-x^3y^2 - 4y^2 - 6x^2) - (-x^3y^2 + 4y^2 - 6x^2)$$

Which of the following expressions is equivalent to the expression shown above?

- ⚠ A) $-8y^2$ Attractor answer with incorrect sign
 B) $8y^2$
 C) $-2x^3y^2 - 12x^2$
 D) $2x^3y^2 + 12x^2$

Keep work neat. When working with problems that require sign changes, it is easy to miss a step.

By simplifying the signs in the second part of the problem, it becomes simple to eliminate many parts of the expression.



2

$$4x^4 - 16x^2y^2 + 16y^4$$

Which of the following expressions is equivalent to the expression shown above?

- A) $(2x^2 - 4y^2)^2$
 B) $(2x - 4y)^4$
 C) $(x^2 - 2y^2)^2$
 D) $(x^2 - 2y^2)^4$

For many students, this problem may be a tough one to factor. It may be faster to try working backwards from the answer choices using FOIL.



3

$$-2(x + 5y)(3x - 3y)$$

Which of the following expressions is equivalent to the expression shown above?

- A) $6(x^2 + 6xy - 5y^2)$
 B) $6x(x^2 + 4xy - 5y^2)$
 C) $6x(-x + y) - 30y(x - y)$
 D) $6x(1 - y) + 30y(x - 1)$

Expand the expression to get:
 $-6x^2 - 24xy + 30y^2$

Instead of expanding all of the answer choices to see if they match, just look to match one term of the expression.

Since only choice C would be able to produce $-6x^2$, it is the correct answer.

HOMEWORK

pg 96, #1-3

pg 100, #21, 24